Problem Set 2

Due November 5

1) Assume that the collision term in the Boltzmann equation of an electron system is of the form

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = -\sum_{\alpha} s_\alpha \int \sigma_\alpha(p,p')dp' \cdot v(p)f(p) + \sum s_\alpha \int \sigma_\alpha(p',p)v(p')f(p')dp'$$

and that the operator D is defined by

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} \equiv -Df = -f(v)\int W(v,v')dv' + \int W(v',v)f(v')dv'$$

where $W(v,v')dv'$ denotes the transition rate of electrons with velocity v into a state with velocity in the interval between $v' + dv'$ (for simplicity, the collisions are assumed to be elastic). Solve the time-dependent Boltzmann equation up through a term linear in E, with the initial condition that E vanishes at $t = -\infty$ and that the electron distribution is the equilibrium distribution f_0, and show that the electric current density at a time t is generally given in the form

$$j_i(t) = \sum_l \int_{-\infty}^{l} E_l(t')dt'\Phi_u(t-t') \qquad (i,l = x,y,z)$$

with

$$\Phi_u(t) = \frac{\langle j_i(t)j_l(0)\rangle}{kT}.$$

Here $\langle\, j_i(t)j_l(0)\,\rangle$ is the correlation function of the electric current, which occurs as a fluctuation in an electron system at equilibrium. Based on this result, express the static and dynamical electric conductivities in terms of the correlation function.

2) Calculate the thermal conductivity χ for the 'usual' case of a hard sphere gas. Do the calculation accurately — do **not** use a krook/crook collision operator.

You may find it helpful to consult "Physical Kinetics" and some posted lecture notes.

Fluids: The problems are intended to give you some short introduction to fluid mechanics. A basic text may be helpful. "Fluid Mechanics" by Landau and Lifshitz is a good one.

3) Show that circulation is conserved for an ideal fluid, where $\mathrm{P} = \mathrm{P}(\rho)$. What does this imply for vorticity $\underline{\omega} = \underline{\nabla} \times \underline{v}$? Derive an equation for $\underline{\omega}$. How does $\underline{\omega}$ evolve for $\mathrm{P} = \mathrm{P}(\rho, T)$?

4) Let $\mathrm{P} = \mathrm{P}(\rho, T)$. Derive the speed of sound for an ideal fluid. Now allow for viscosity and thermal diffusion. Calculate the dispersion relation. Calculate the evanescence length for a wave with wave number k.

5) Calculate the pressure for a viscous incompressible fluid.

6) An incompressible fluid rotates with $\underline{\Omega} = \Omega(r)\hat{z}$. Calculate the wave dispersion relation. For what profile $\Omega(r)$ does instability result?

7) Use the Kubo Formula to calculate the thermal diffusivity of a hard sphere gas. Compare your answer to that obtained by the Chapman–Enskog method, for short mean free path. A krook model approach is OK. What can be said about the correlation time in that limit?

8) Consider a simple system with kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{\mathrm{q}}{m}E\frac{\partial f}{\partial v} + \frac{\mathrm{q}}{m}E_{ext}(x,t)\frac{\partial f}{\partial v} = c(f)$$

E satisfies Poisson's equation.

Take f_o Maxwellian, formed by a very slow collisional process. $c(f)$ is negligible on dynamical time scales. $E_{ext}(x,t)$ varies slowly in space and time.

i) Compute the linear response δf to E_{ext}. What determines the fluctuating current correlation time?

ii) Use δf to derive a mean field evolution equation for f_o, on $t < \tau_{coll}$.

iii) What physics determines the evolution of f_o?

9) i) Calculate the mobility of a Brownian particle $\mu(\omega)$ using the Kubo formalism.

ii) Continue the cumulant expansion of the function F derived in class. Calculate the correlation to diffusion. What does it depend on?

10) Read Onsager's paper on Reciprocity posted. Write a one-page (typed) abstract of the paper, in **your own words**. Address:

— physics motivation
— key results
— exceptions, complications